Optical Phase Shifts and the Nightmare of Conventions

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Reflected light intensity varies with angle of incidence \((\theta_i)\).
Glass windows act like a mirror when it is dark outside.
Fresnel Equations express transmission and reflection amplitude coefficients.

\[ t = \frac{E_t}{E_i} \quad \text{transmitted electric field amplitude} \]

\[ r = \frac{E_r}{E_i} \quad \text{reflected electric field amplitude} \]
Reflected light intensity varies with the polarization of the incident beam.
Fresnel Equations are derived from Maxwell’s Equations at a boundary.

**Transmitted Light:**

\[
\begin{align*}
t_s(\theta_i, n_{ti}) &= \frac{2 \cos \theta_i}{\cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}}, \\
t_p(\theta_i, n_{ti}) &= \frac{2 \cos \theta_i}{n_{ti} \cos \theta_i + \sqrt{1 - \frac{\sin^2 \theta_i}{n_{ti}^2}}},
\end{align*}
\]

**Reflected Light:**

\[
\begin{align*}
r_s(\theta_i, n_{ti}) &= \frac{\cos \theta_i - \sqrt{n_{ti}^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}}, \\
r_p(\theta_i, n_{ti}) &= \frac{n_{ti}^2 \cos \theta_i - \sqrt{n_{ti}^2 - \sin^2 \theta_i}}{n_{ti}^2 \cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}}.
\end{align*}
\]
Reflected field amplitude and phase depend on the **polarization** of the incident field, angle of incidence, and refractive indices.
Polarization state changes upon reflection.
Polarization state upon total internal reflection becomes elliptical.
Hecht vs. Pedrotti on phase shifts upon reflection
Hecht vs. Pedrotti

on phase shifts upon external reflection

(a) TE mode, external reflection (air to glass)

(b) TM mode, external reflection (air-to-glass)
Assumed direction for the reflected field matters.

If $E_r$ is in $+y_r$ direction, $r_s < 0$  
If $E_r$ is in $-y_r$ direction, $r_s > 0$
Fresnel vs. Verdet Convention

• Phase shift is to be understood in reference to the assumed direction for the reflected field, not in reference to the incident field.
• Fresnel: \( r_s = r_p \) at **normal** incidence
• Verdet: \( r_s = r_p \) at **glancing** incidence
Hecht vs. Pedrotti

on phase shifts upon internal reflection

(b) TE mode, internal reflection (glass to air)

(d) TM mode, internal reflection (glass-to-air)
Hecht vs. Salik
Phase difference \((\delta_p - \delta_s)\) affects the polarization state.
Phase-difference matters for the polarization state of the reflected beam.
Is it the ambiguity in the square root of the complex numbers?

\[ r_s(\theta_i, n_{ti}) = \frac{\cos \theta_i - n_{ti} \cos \theta_t}{\cos \theta_i + n_{ti} \cos \theta_t} \]

Complex for total internal reflection

The physical solution

\[ \sqrt{-1} = \begin{cases} +i \\ -i \end{cases} \]
Writing the reflected field

\[ E_i = A \cos(kz - \omega t) = A \Re \left[ e^{i(kz - \omega t)} \right] \Rightarrow \tilde{E}_i = Ae^{i(kz - \omega t)} \]

\[ \tilde{E}_r = rAe^{i(kz - \omega t)} = Ae^{i\delta}e^{i(kz - \omega t)} \Rightarrow E_r = A \cos(kz - \omega t + \delta) \]

Reflected Field Components:

\[ E_x = A \cos(kz - \omega t + \delta_p) \]

\[ E_y = A \cos(kz - \omega t + \delta_s) \]
How you express the fields matters.

\[ E_x = A \cos(kz - \omega t + \delta_p) \quad E_y = A \cos(kz - \omega t + \delta_s) \]

\[ \theta_i = 60^\circ \]
\[ n_i = 1.50 \]
\[ n_t = 1.00 \]
\[ \delta_p = -136.2^\circ \]
\[ \delta_s = -95.7^\circ \]
How you express the fields matters.

\[ E_x = A \cos(\omega t - kz + \delta_p) \quad E_y = A \cos(\omega t - kz + \delta_s) \]

\[ \theta_i = 60^\circ \]
\[ n_i = 1.50 \]
\[ n_t = 1.00 \]
\[ \delta_p = -136.2^\circ \]
\[ \delta_s = -95.7^\circ \]
How you express the fields matters.

\[ E_x = A \cos\left(\omega t - kz - \delta_p\right) \quad E_y = A \cos\left(\omega t - kz - \delta_s\right) \]
Conclusions

• Fresnel reflections can be utilized to emphasize the importance of conventions in Physics.
• Textbooks give us an opportunity to discuss how different conventions can lead to seemingly conflicting results.

Quantitative investigation of Fresnel reflection coefficients by polarimetry

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