# ACTIVITIES TO ILLUSTRATE THE STATISTICAL NATURE OF ENTROPY

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"Ludwig Boltzmann, who spent much of his life studying statistical mechanics, died in 1906, by his own hand. Paul Ehrenfest, carrying on the work, died similarly in 1933. Now it is our turn to study statistical mechanics. Perhaps it will be wise to approach the subject cautiously."

- from *States of Matter* by David L. Goodstein

# Inspiration

Many of the ideas presented here originated with:

- Tom Moore (Pomona College)
- Harvey Leff (Cal Poly Pomona)
- Frank Lambert (Occidental College)

# Teachers explain entropy– 1

*How do you explain what entropy is, both to yourself and to your students?*

- "Entropy is the move from order to disorder, caused by the unavailability of energy in a system that would keep the system in an ordered state."
- "Entropy to me is a natural movement of a system from order to disorder. Order takes energy to maintain. I like to think about it like my desk at work. My desk naturally transitions from well-ordered and clean to chaotic and messy."


DEPT. OF
ENTROPY

# Teachers explain entropy– 2

- “To students (depending on the level) I often emphasis how entropy gives a "direction" to the way things happen. I usually steer clear of words like "order" as this gets a bit dicey in my opinion.”
- “I like to use videos running backwards and asking how they know the videos are running backwards. This involves the "arrow of time" idea and that leads to entropy.”

# Teachers explain entropy– 3

- “Depends on the level. In intro courses, if they ask I tell them that it's a measure of how random a system is, but to wait for the details when they take upper-level Thermo.”

# Textbooks explain entropy– 1

*Conceptual Physics* by Hewett (8th edition)

- "Natural systems tend to proceed toward a state of greater disorder."
- "Entropy is the measure of the amount of disorder."
- "By disordered, we mean more random."
- "You finally straighten up your room. Everything is dust-free and it's proper place. A week later it is cluttered again."

- "The second law qualifies this [conservation of energy] by adding that in transformations, energy "deteriorates" from more useful forms to less useful forms. Energy becomes more diffuse and ultimately degenerates into waste."

# Textbooks explain entropy– 2

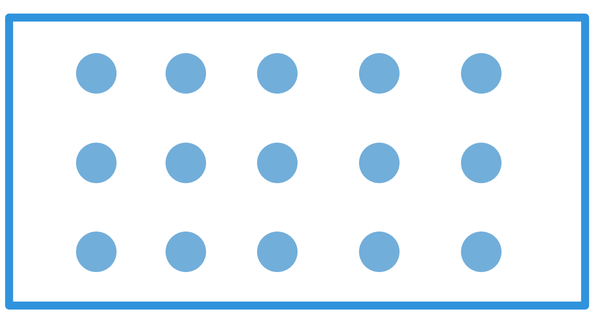
*Physics: Principles with Applications* by Giancoli (5th edition)

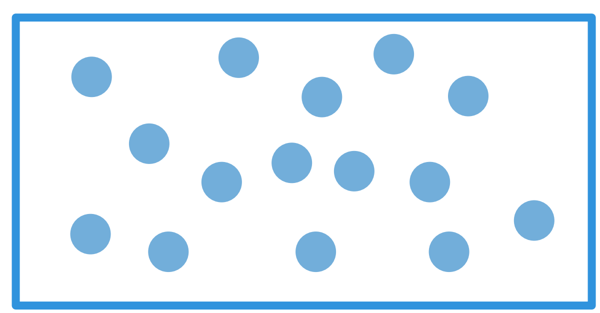
- “To get a feel for the concept of entropy, we can relate it to the concepts of order and disorder. In fact, the entropy of a system can be considered a measure of the disorder of the system.”
- “Exactly what we mean by disorder may not always be clear,… ”
- “When a hot object is put in contact with a cold object, heat flows from the high temperature to the low until the two objects reach the same in intermediate temperature. At the beginning of the process we can distinguish two classes of molecules: those with a high average connect energy and those with low average connect energy. After the process, all of the molecules are in one class with the same average kinetic energy, and we no longer have the more orderly arrangement of molecules in two classes. Order has gone to disorder.”

# Textbooks explain entropy– 3

*College Physics by Knight, Jones and Field* (3rd edition)

- "Scientists and engineers use the term entropy to quantify the probability that a certain state of a system will occur."





# About disorder…

There are several faults entropy as disorder:

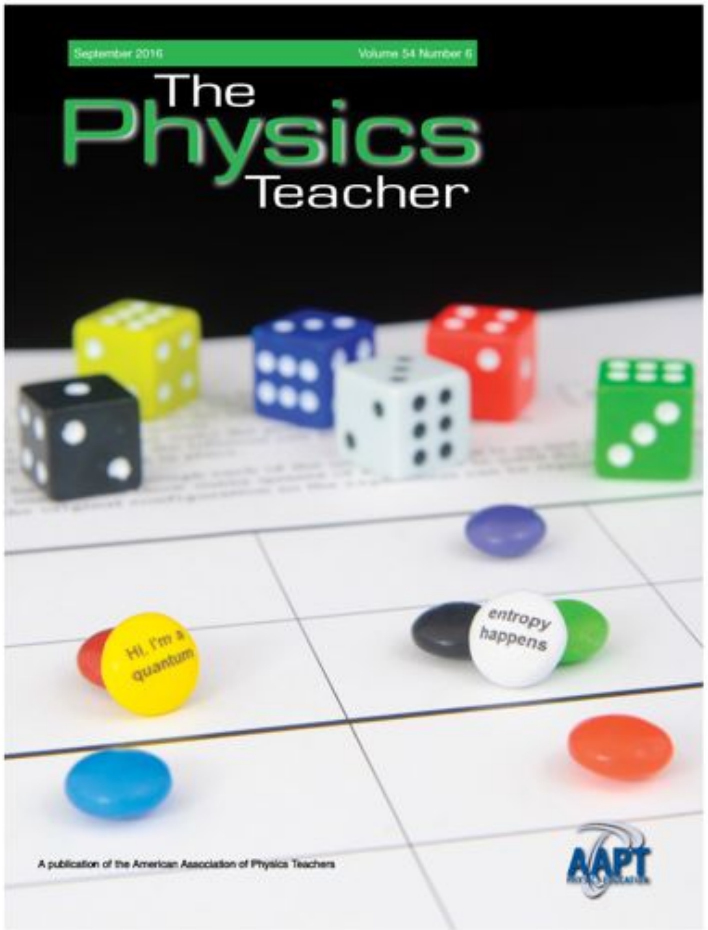
- Disorder is a subjective description that has no rigorous definition.
- Disorder is nearly always interpreted as a comment on the arrangement of a single microstate, yet entropy applies to a macrostate, not a microstate.

# Goals of activities

- Explain the probabilistic nature of second law
- Describe the role of conservation of energy at the microscopic level
- Explain the differences between microstates and macrostates
- Describe entropy in language that does not include “disorder”
- Predict how non-equilibrium systems will evolve
- Articulate why hot objects cool

# Activities described in *TPT*

- September 2016
- "The Macro and Micro of it Is that Entropy Is the Spread of Energy"
- http://dx.doi.org/10.1119/1.4961175



# Tinkering with a "toy" model– 1

- First develop the language of microstates and macrostates
- The total number of markers in A (A1 or A2) is called a *macrostate*- it describes the system as a whole and ignores the "microscopic" details.
- A *microstate* does know what is going on at the small scale- it gives you the information about each box.
- The *multiplicity* of a macrostate is the number of microstates that have the same macrostate.

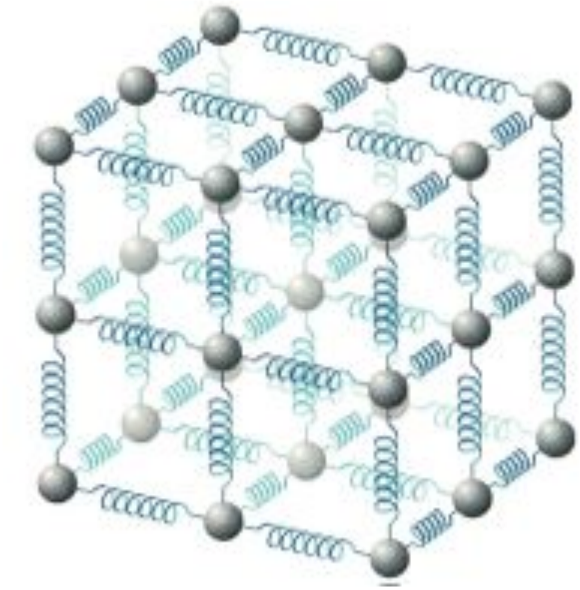
# Tinkering with a “toy” model– 2

The number of microstates, multiplicity, for each possible macrostate of our toy system. The most likely macrostate is the one with the greatest multiplicity.

| $E_A$ (energy in solid A) | W (multiplicity) |
|---|---|
| 0 | 5 |
| 1 | 8 |
| 2 | 9 |
| 3 | 8 |
| 4 | 5 |

# Defining the “toy”– 1

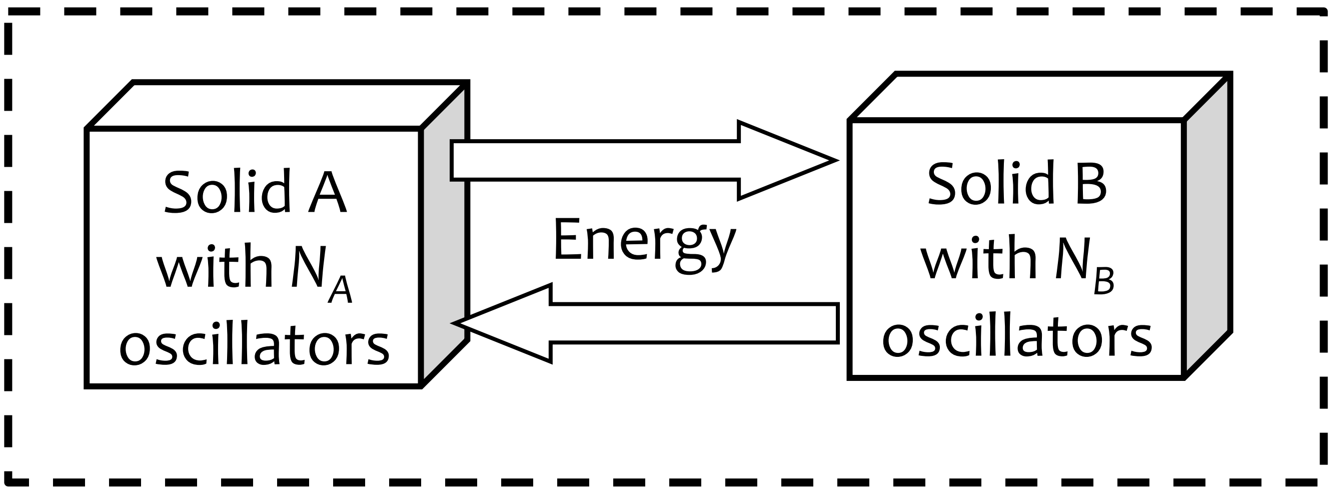
- We’re considering two Einstein solids in thermal contact with each other
- These bonds are modeled as simple harmonic oscillators => evenly-spaced energy levels.



- Total energy in any bond is simply an integral multiple of *hf*, where *h* is Planck’s constant and *f* the natural frequency of the oscillator.

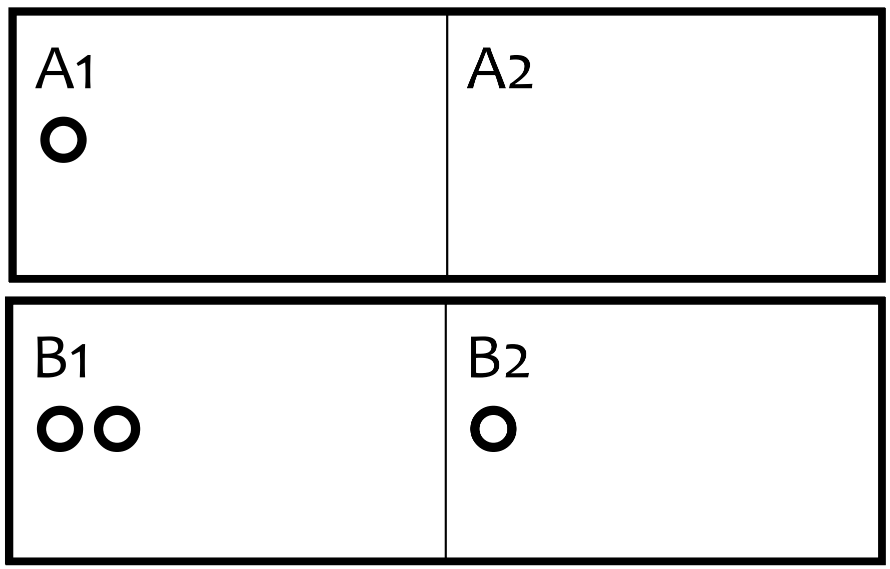
# Defining the “toy”– 2

- Energy is conserved within the isolated system



# Time evolution– 1

- Markers (quantum of energy) can move freely between boxes (oscillators).

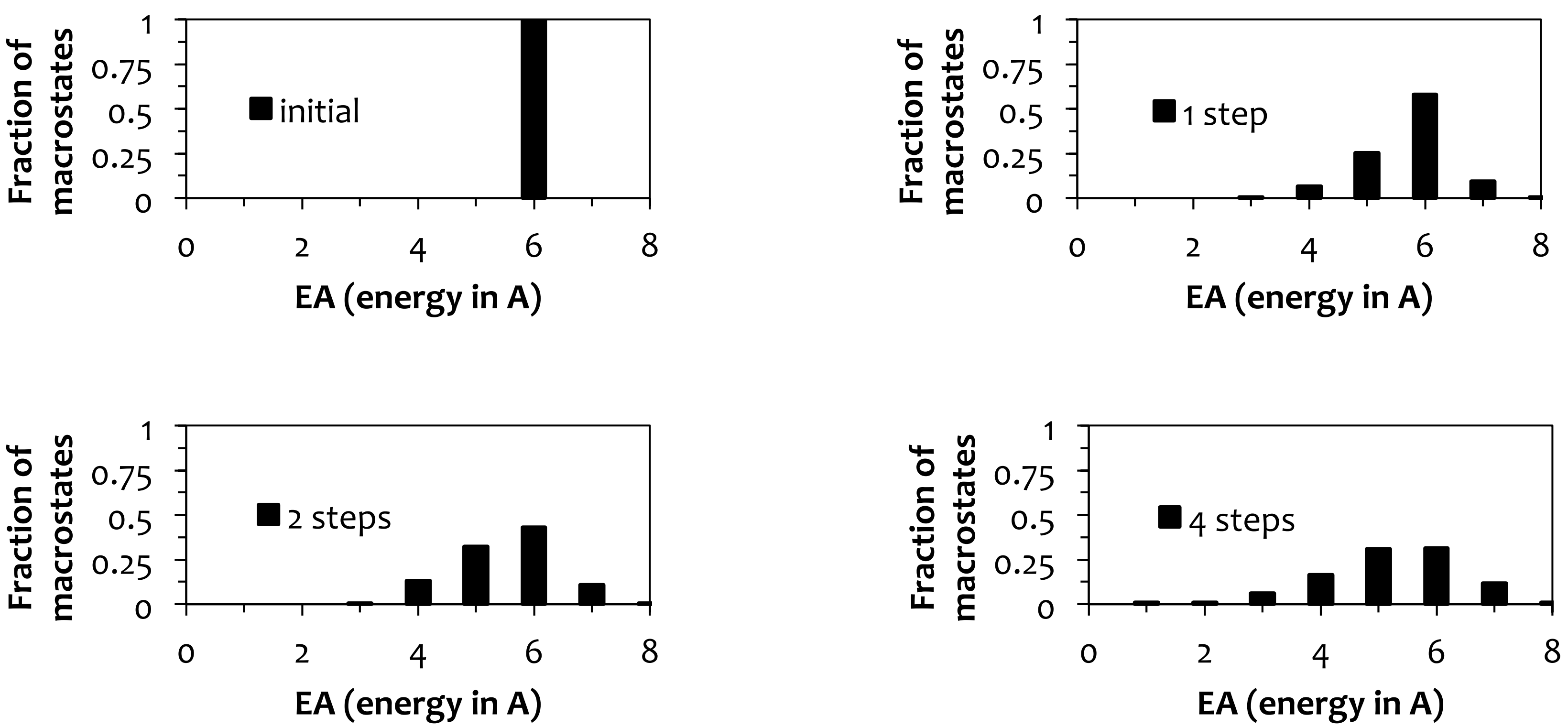


# Guidelines for class

- Each student begins with the same macrostate, but a different microstate
- After each time evolution (one to four “step”), each student resets their system back to his/ her original microstate.
- By repeating the time evolutions multiple times, the class ensemble can better see the irreversible nature

# Distributions of macrostates

- Thirty-two different initial microstates were followed through ten different time evolutions, yielding 320 separate simulations.



# Entropy

- The Boltzmann expression for entropy is:
  - $S = k_B \ln W$
- Yes, but what does this mean? What physical intuition does it provide?
- *S* (entropy) can be thought of as an abbreviation for “spread of energy”.
- Energy spreads not only spatially, but throughout all of phase space to all degrees of freedom.

# Conclusions

- These activities help students learn about the abstract concepts of energy and entropy.
- The first guides students through the foundational ideas of microstates and macrostates.
- The second illustrates how systems tend to evolve toward macrostates in which the energy is spread across all degrees of freedom.
- Due their large multiplicities, larger systems are best investigated via computer simulations.

# Thanks!

- Jeff Phillips (jphillips@lmu.edu)
- *TPT* article: http://dx.doi.org/10.1119/1.4961175
- This presentation and sample worksheets will be posted to the SCAAPT meeting page.